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Interferometry and coherence of nonstationary light

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We consider temporally integrating interferometric measurements and their relation to the coherence properties of nonstationary light. We find that performing such experiments as a function of time delay is equivalent to spectrally resolving the interference patterns, and time domain coherence information can be obtained from field autocorrelation only if the source is of the Schell-model type. In analogy to autocorrelation, we introduce field cross-correlation, which can be used to determine the complete complex field of unknown signal pulses if suitable probe pulses are available. We demonstrate our findings with simulated supercontinuum and free-electron laser ensembles, and discuss the prospect of carrying out experiments.

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Temporal coherence of pulsed fields is of great interest to the scientific community, with an increasing supply of light sources for different applications. Variable delay Michelson's interferometer has conventionally been employed to measure the temporal coherence of stationary light [1]; however, for nonstationary pulsed light the situation is more involved. An important directly measurable coherence quantity for pulsed light is the Dudley–Coen degree of coherence [2, 3]. In an experimental setting it is obtained by interfering consecutive pulses with a modified Michelson interferometer and spectrally resolving the fringe pattern. This method characterizes the quasi-coherent part of the spectral degree of coherence, and it has been employed to assess the coherence of supercontinuum light [3–6], although it has limitations due to requiring two consecutive pulses and spectral resolution.

A general method to quantify the degree of temporal coherence of a train of pulses is to measure the amplitude and the phase of the field in a single-shot manner, for instance with a FROG [7] or a SPIDER [8]. Afterwards, one can compute the mutual coherence function (MCF) and the cross-spectral density (CSD) from the data, given that the measurements are done over a sufficiently large ensemble of pulses. This approach is applicable to a variety of sources, mainly in the domain of visible light. Since nonlinear materials in shorter wavelengths are scarce, these methods cannot be applied easily to synchrotron or free-electron laser (FEL) radiation.

Recently, it was proposed that estimating the temporal degree of coherence and the pulse length from field autocorrelation measurements is possible, if single-shot interference patterns can be recorded. The idea is to compare two differently ensemble-averaged autocorrelation traces: one where the phase is kept throughout averaging, and one where the phase is removed prior to it. It is then concluded that the pulse length and coherence time can be found with such measurements [9]. This method was employed for the estimation of FEL-pulse characteristics [10], and an in-depth analysis of the scheme has also been carried out for a particular type of pulses [11, 12]. In the present study, we examine the applicability of such interferometric methods for general pulsed fields and expand the possible measurements onto field cross-correlations.

We represent the random light field by the complex analytic signal $E(t)$ and its Fourier transform $E(\omega)$, i.e.,

$$E(t) = \mathcal{F}\{E(\omega)\} = \frac{1}{2\pi}\int_0^\infty E(\omega)\exp(-i\omega t)\,\mathrm{d}\omega, \quad (1)$$

where the lower limit is zero due to analyticity. The temporal coherence properties of nonstationary light can then be quantified by means of the MCF in the time domain, and the CSD in the spectral domain, defined in the average $(\bar{t}, \bar{\omega})$ and difference $(\Delta t, \Delta\omega)$ coordinates as [13]

$$\Gamma(\bar{t}, \Delta t) = \langle E^*(\bar{t} - \Delta t/2)E(\bar{t} + \Delta t/2)\rangle, \quad (2)$$

$$W(\bar{\omega}, \Delta\omega) = \langle E^*(\bar{\omega} - \Delta\omega/2)E(\bar{\omega} + \Delta\omega/2)\rangle, \quad (3)$$

respectively, where the asterisk denotes complex conjugation and the angle brackets stand for ensemble averaging over the pulse train. These functions contain all the coherence information of the pulsed field, including the intensity $\Gamma(\bar{t}, 0) = I(\bar{t})$ and the spectral density $W(\bar{\omega}, 0) = S(\bar{\omega})$. The MCF and CSD can be normalized, yielding the complex degrees of coherence

$$\gamma(\bar{t}, \Delta t) = \frac{\Gamma(\bar{t}, \Delta t)}{\sqrt{I(\bar{t} - \Delta t/2)I(\bar{t} + \Delta t/2)}}, \quad (4)$$

$$\mu(\bar{\omega}, \Delta\omega) = \frac{W(\bar{\omega}, \Delta\omega)}{\sqrt{S(\bar{\omega} - \Delta\omega/2)S(\bar{\omega} + \Delta\omega/2)}}. \quad (5)$$

These correlation functions contain a great amount of information, but there is no known general method to directly measure them, the best way being single-shot pulse shape measurements over a large ensemble of pulses. In some cases this is impractical or even impossible, and therefore we will consider experimentally simpler schemes.

We shall begin from the simplest case, the variable delay Michelson interferometer. It measures the field autocorrelation $A(\Delta t)$ of time-domain pulses $E(t)$ as in

$$A(\Delta t) = \int_{-\infty}^{\infty} E^*(t - \Delta t/2)E(t + \Delta t/2)\mathrm{d}t. \quad (6)$$

Inserting from Eq. (1) into Eq. (6) and applying straightforward calculus, leads to an expression in terms of the spectral amplitudes $E(\omega)$

$$A(\Delta t) = \mathcal{F}\{|E(\omega)|^2\}. \quad (7)$$

This result is the well-known autocorrelation theorem, and it has profound consequences in Michelson interferometry. One might intuitively think that the field autocorrelation is directly related to the temporal pulse shape and duration, but this is in fact not the case. The autocorrelation theorem of Eq. (7) holds for any spectral phase, and as such it is insensitive to changes in the pulse shape and length, given the spectrum does not change. Actually, field interferometric techniques in the temporal domain yield only spectral domain information, and vice versa. This is in contrast to the widely employed intensity autocorrelation, which does provide some estimate of the pulse length, but it is a nonlinear measurement [13]. However, if additional knowledge on source characteristics is available, spectral measurements can yield time-domain coherence information, as we will see later. All of the following temporal field correlation measurements have spectral domain counterparts, but components with different frequencies produce a rapidly moving interference pattern, which cannot be measured with modern detectors. Therefore we will not discuss them here.

A detector faster than the repetition rate of the laser would enable to record the autocorrelation of each pulse separately. Ensemble averaging the resulting single-shot traces yields

$$\langle A(\Delta t)\rangle = \mathcal{F}\{\langle|E(\omega)|^2\rangle\} = \mathcal{F}\{S(\omega)\}. \quad (8)$$

Such a measurement contains information on the power spectrum of the source, which is sufficient for evaluating the coherence time of stationary and quasi-stationary light [14]. In fact, comparing Eqs. (2) and (6) reveals that the ensemble averaged field autocorrelation equals the time integrated MCF, viz.,

$$\langle A(\Delta t)\rangle = \int_{-\infty}^{\infty} \Gamma(t, \Delta t)\mathrm{d}t. \quad (9)$$

This, in turn, indicates that $\langle A(\Delta t)\rangle$ provides the coherence correctly for sources that are of the Schell-model type, i.e., sources whose correlation function varies only along the time delay axis Δt. However, from the measurements alone there is no way of knowing whether a source has this property. In the case of more coherent light, the measurement becomes problematic. For example, for a completely coherent pulse train, a field autocorrelation measurement always yields a finite coherence time corresponding to the Fourier transform of the power spectrum (i.e., transform-limited pulse duration).

We can employ the method proposed in [9] to find indications of reduced coherence from single-pulse autocorrelation measurements, even if the source is not of the Schell-model type. Discarding the phase before ensemble averaging, we get

$$\langle|A(\Delta t)|\rangle = \langle|\mathcal{F}\{|E(\omega)|^2\}|\rangle. \quad (10)$$

If the train of pulses is completely coherent, then necessarily $\langle|A(\Delta t)|\rangle = |\langle A(\Delta t)\rangle|$. For a pulse train that features spectral amplitude fluctuations, $\langle|A(\Delta t)|\rangle$ displays a pedestal in addition to the autocorrelation peak [9, 11]. Consequently, any difference between Eqs. (9) and (10) indicates reduced coherence, but the degree of coherence cannot be numerically estimated. Due to Eq. (7), this method is blind to spectral phase and cannot measure changes in the pulse length which arise from spectral phase variations. This also means that it cannot discern a partially coherent pulse train from a completely coherent one, if the amplitudes of the Fourier spectra for the individual pulses stay constant. Hence, it is desirable to look for techniques which yield more information on the properties of pulse trains.

Let us take two different time domain pulses, $E_i(t)$ and $E_j(t)$, where $i \neq j$. Correlating these in an interferometer results in the field cross-correlation

$$X(\Delta t) = \int_{-\infty}^{\infty} E_i^*(t - \Delta t/2)E_j(t + \Delta t/2)\mathrm{d}t. \quad (11)$$

As in Eqs. (6) and (7), we find that the field cross-correlation is the Fourier transform of the interfering Fourier components

$$X(\Delta t) = \mathcal{F}\{E_i^*(\omega)E_j(\omega)\}. \quad (12)$$

This result is the cross-correlation theorem. Unlike the autocorrelation theorem, it retains spectral phase and thus may yield more information on the field. In the best-case scenario, the complete complex fields of unknown signal pulses can be retrieved if one has known and highly coherent probe pulses. This is achieved, for example, when a stable pulse train from a mode-locked laser with known output is split into two and one of the trains is passed through some dynamic system. Then the original pulses can be used to probe the modulated signal pulses. Taking the pulses $E_i(\omega)$ as the probe and $E_j(\omega)$ as the signal, we can straightforwardly find the signal pulses from Eq. (12), and hence construct the associated complex correlation functions. This allows us to assess the effect of the optical system on pulse coherence. The novel cross-correlation measurement is effectively spectral interferometry without a spectrograph, enabling one to find the amplitude and phase of the individual signal pulses.

It is possible that the pulse repetition rate is too high to perform single-shot measurements in this configuration, in which case it is necessary to look at what information can be retrieved from the ensemble average $\langle X(\Delta t)\rangle$. If we take the probe pulse train $E_i(\omega)$ as completely coherent, we can remove it from averaging and obtain

$$\langle X(\Delta t)\rangle = \mathcal{F}\{E_i^*(\omega)\langle E_j(\omega)\rangle\}, \quad (13)$$

implying that the ensemble averaged cross-correlation between the probe and signal pulses can be separated into the coherent probe and the mean signal field. If the mean signal is separable to quasi-coherent and quasi-stationary contributions, then $\langle X(\Delta t)\rangle$ can be used to construct the quasi-coherent part of the CSD [3]. Sources with this property include supercontinua (SC) generated in nonlinear fibers, which we will examine next.

Assume that $E_i(t)$ and $E_j(t)$ are two different pulses from a fiber-generated SC source. If the source produces N different pulse realizations, we can pick $N^2 - N$ pairs from that group.

Considering all possible combinations, we may write the ensemble averaged cross-correlation as

$$\langle X(\Delta t)\rangle = \frac{1}{N^2-N}\mathcal{F}\left\{\sum_{i\neq j}^{N} E_i^*(\omega)E_j(\omega)\right\}. \quad \textbf{(14)}$$

Since we are summing over all pairs, the quantity in the curly brackets is real. Including all pairings into the cross-correlation is not realistic in practice. However, the imaginary part of the function decreases rapidly and picking sufficiently many pulse pairs from the train leads to a good approximation of Eq. (14). The cross-correlation can further be expressed as

$$\langle X(\Delta t)\rangle = \frac{1}{N^2-N}\mathcal{F}\left\{|\sum_{i=1}^{N} E_i(\omega)|^2 - \sum_{i=1}^{N}|E_i(\omega)|^2\right\}, \quad \textbf{(15)}$$

which can be simplified by introducing the ensemble averages

$$\langle X(\Delta t)\rangle = \frac{1}{N-1}\mathcal{F}\left\{N|\langle E(\omega)\rangle|^2 - \langle|E(\omega)|^2\rangle\right\}. \quad \textbf{(16)}$$

Performing this measurement over a large ensemble of pulses, the last term becomes insignificant and we may approximate

$$\langle X(\Delta t)\rangle \approx \mathcal{F}\{S_{\mathrm{qc}}(\omega)\}, \quad \textbf{(17)}$$

where $S_{\mathrm{qc}} = |\langle E(\omega)\rangle|^2$ is the quasi-coherent part of the spectrum [3]. A field cross-correlation performed in this manner yields the same result as a modified Michelson interferometer used to measure the Dudley–Coen degree of coherence [3, 5]. But the spectral resolution is introduced via temporal integration rather than a spectrograph, greatly simplifying the setup and enabling the measurement of weaker trains.

In order to illustrate the quality of data one may obtain from measurements outlined above, we shall next turn to simulated ensembles of SC and FEL pulses, starting with the former. We employ the definitions in Eqs. (4) and (5) to quantify the coherence properties of the pulse ensembles, and compare the results to the quantities obtained with relatively simple interferometric experiments. We used 1000 SC realizations that were generated by numerically solving the generalized nonlinear Schrödinger equation, details of which can be found elsewhere [15]. The ensemble simulates a case in which 1060 nm and 1 ps pump pulses are injected into a 20 cm long anomalously dispersive fiber, resulting in a very low coherence pulse train [3]. The absolute values of the complex degrees of coherence are given in Fig. 1, together with the normalized average intensity and spectral density. The solid blue line denotes the complete intensity and spectrum, whereas the dashed red line is the quasi-coherent part obtained by inverse Fourier transforming Eq. (17). Furthermore, we have plotted the normalized temporal autocorrelation traces obtained with both methods in Fig. 2, together with the normalized temporal cross-correlation trace of Eq. (17). It needs to be noted that unlike autocorrelation, the cross-correlation function is not necessarily symmetric with respect to origin. It becomes symmetric only in two cases: either when the pulse train is completely coherent, or when the ensemble is extended to infinity.

Figure 1 shows that the SC ensemble in this particular case is nearly incoherent, with most of the spectrum dominated by the quasi-stationary contribution (spectral region outside the quasi-coherent peak). However, an autocorrelation measurement would not yield a correct value for the coherence time, since the SC radiation is clearly not of the Schell-model type. In Fig. 2, we see how the different autocorrelations behave. A

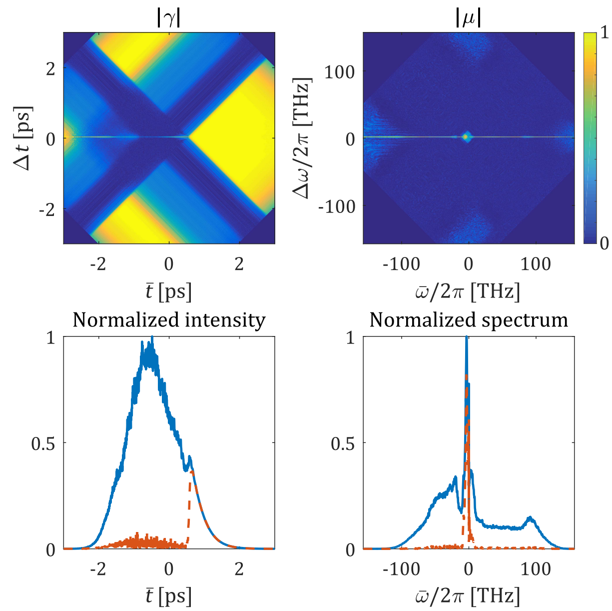

Fig. 1. Coherence properties of the SC pulses. Upper row: absolute values of the degrees of coherence in temporal and spectral domains. Lower row: normalized intensity and spectrum (solid blue lines), and the related quasi-coherent contributions (dashed red lines). The average time $\bar{t}$ is in the moving pulse reference frame and the frequency $\bar{\omega}$ is with respect to the center frequency $\omega_0/2\pi = 283$ THz.

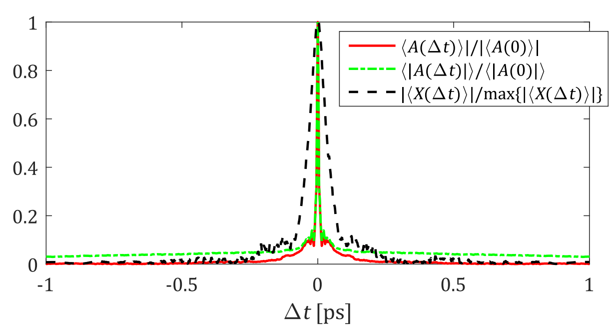

Fig. 2. Normalized correlation traces for the SC source. Autocorrelation with phase (solid red line), autocorrelation without phase (dash-dotted green line), and cross-correlation (dashed black line).

pedestal is formed when the phase is discarded prior to ensemble averaging, which is a manifestation of decreased coherence. The cross-correlation is also clearly different from the two autocorrelation traces, though it is almost symmetric as well.

We then turn to a demonstration using FEL pulses. The FEL ensemble was created with a dedicated program called Genesis [16], which is a time-dependent simulation software that solves the 3D FEL equations and yields a complete complex representation of the FEL pulses. Genesis sub-samples the electron phase-space by means of macroparticles and discretizes the electromagnetic field – which is calculated at different positions inside the FEL undulator – along a transverse grid and in temporal slices. For this paper, an ensemble of 100 pulses with a central wavelength of 0.15 nm were simulated using Genesis, based on electron start-to-end simulations for a 17.5 GeV, 20 pC electron beam in the linear regime at SASE1 undulator of the

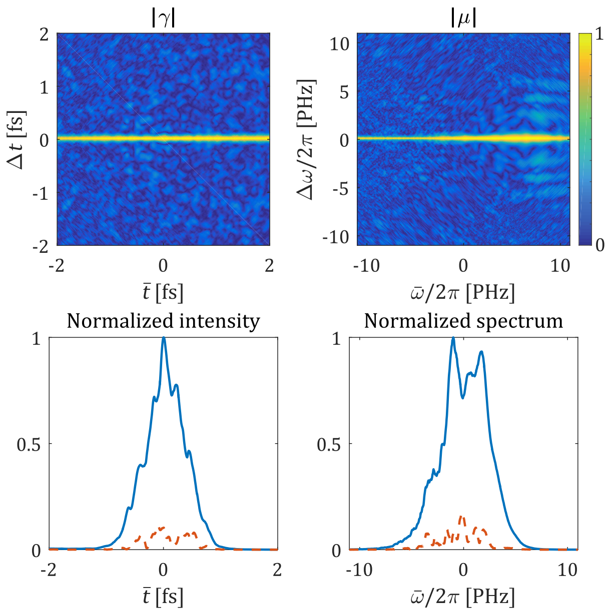

Fig. 3. Same as Fig. 1, but for an FEL pulse ensemble. The center frequency for these pulses is $\omega_0/2\pi = 2$ EHz.

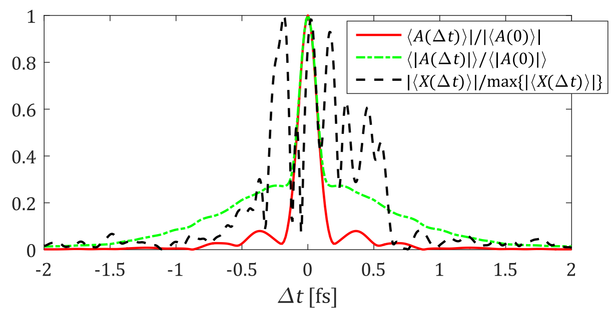

Fig. 4. Same as Fig. 2, but for an FEL pulse ensemble.

European X-ray free-electron laser (European XFEL) [17].

These particular FEL pulses are almost of the Schell-model type, as can be seen from Fig. 3, and now the coherence time may be estimated via field autocorrelation. In addition, the pulse train is highly quasi-stationary, with nearly negligible quasi-coherent contribution. SASE1 is based on self-amplified spontaneous emission (SASE), that is, the electron shot noise is amplified as the relativistic electron beam travels through the FEL undulator. This mechanism intrinsically leads to fluctuations from pulse to pulse due to which SASE based FELs typically have low spectral and temporal coherence, if several longitudinal modes are allowed to oscillate. By tuning the center wavelength and the electron pulse duration, it is possible to control the number of statistically independent modes in the pulse [18], in which case the field autocorrelation no longer is enough to estimate the coherence time of the field, as was discussed after Eq. (9). In Fig. 4, we can see a more pronounced pedestal than in the case of SC, because the FEL radiation does not have a strong coherent peak in the spectrum. The cross-correlation trace is found to be highly oscillating when compared to the corresponding SC quantity. This is caused by the smaller ensemble of FEL pulses – which was limited mainly by the large size of the data – as well as the low degree of coherence of the pulse train.

In conclusion, we have examined different types of field interference measurement schemes and their properties. The usual field autocorrelation in general gives only information on the spectrum, but it can be used to estimate the coherence time, provided additional evidence is available that the source is of the Schell-model type. Discarding the phase and then performing an ensemble average over the autocorrelation traces can yield some qualitative knowledge on the stability of the pulse train, since the pedestal appears when coherence is low. As we have demonstrated, there exist field interference experiments that can provide markedly more information, namely the variations of field cross-correlation. Such measurements produce more information due to the cross-correlation theorem retaining the spectral phase, but they are more limited in applicability than the field autocorrelation measurements. The most limiting factor when considering, for instance, FEL pulses, could be the relatively low repetition rate, which would necessitate the use of a very long delay line. However, as proposed in [19] and first tested in [20], a magnetic chicane between two undulator parts – which will soon be available at the SASE2 line of the European XFEL – can be used to measure the cross-correlation, at least in the case when no major energy chirp is present in the electron beam. This may open up the possibility to characterize the coherence properties of FEL radiation in detail.

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